The Contextual Bandits Problem Techniques for Learning to Make High-Reward Decisions

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- repeat:
 - 1. website visited by user (with profile, browsing history, etc.)
 - 2. website chooses ad/content to present to user
 - 3. user responds (clicks, leaves page, etc.)
- goal: make choices that elicit desired user behavior

- repeat:
 - 1. doctor visited by patient (with symptoms, test results, etc.)
 - 2. doctor chooses treatment
 - 3. patient responds (recovers, gets worse, etc.)
- goal: make choices that maximize favorable outcomes

The Contextual Bandits Problem

• repeat:

- 1. learner presented with context
- 2. learner chooses an action
- 3. learner observes reward (but only for chosen action)
- goal: learn to choose actions to maximize rewards
- general and fundamental problem: how to learn to make intelligent decisions through experience

<u>Issues</u>

- classic dilemma:
 - exploit what has already been learned
 - explore to learn which behaviors give best results
- in addition, must use context effectively
 - many choices of behavior possible
 - may never see same context twice need to generalize
- selection bias: if explore while exploiting, will tend to get highly skewed data
- efficiency

- overview of some of the algorithms and techniques used for contextual bandits (and variants)
- want algorithms that:
 - are general-purpose and practical fast and simple to implement
 - can learn complex behaviors based on context
 - have provably strong statistical guarantees

Outline

- formalizing the learning problem
- algorithms

• an application and a next step

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Formal Model

• repeat, for $t = 1, \ldots, T$:

1a. learner observes context x_t

1b. reward vector $\mathbf{r}_t \in [0, 1]^K$ chosen (but not observed)

- 2. learner selects action $a_t \in \{1, \ldots, K\}$
- 3. learner receives observed reward $r_t(a_t)$
- goal: maximize total reward:

$$\sum_{t=1}^{T} r_t(a_t)$$

• for now: assume pairs (x_t, \mathbf{r}_t) chosen at random i.i.d.

Example

	Actions		
Context	1	2	3
(<i>Male</i> , 50,)	1.0	0.2	0.0
$(Female, 18, \ldots)$	1.0	0.0	1.0
($\mathit{Female}, 48, \ldots$)	0.5	0.1	0.7
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total reward = $0.2 + 1.0 + 0.1 + \cdots$

Policies

- aim: learn to choose actions based on context
- want good policy: rule for selecting action from context
- e.g.:
 - $\begin{array}{ll} \mbox{If (sex = male)} & \mbox{choose action 2} \\ \mbox{Else if (age > 45)} & \mbox{choose action 1} \\ \mbox{else} & \mbox{choose action 3} \end{array}$
- policy π : (context x) \mapsto (action a)
- before learning, must choose general form of policies to be used
 - \Rightarrow defines policy space Π
 - e.g.: all decision trees (nested "if-then-else" rules)
 - tacit assumption:
 ∃ (unknown) policy π ∈ Π that gives high rewards

Learning with Context and Policies

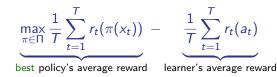
- goal: learn through experimentation to do (almost) as well as best $\pi \in \Pi$
- assume ∏ finite, but typically extremely large
- policies may be very complex and expressive ⇒ powerful approach
- challenges:
 - Π extremely large
 - need to be learning about all policies simultaneously while also performing as well as the best
 - when action selected, only observe reward for policies that would have chosen same action
 - exploration versus exploitation on a gigantic scale!

Formal Model (revisited)

• repeat, for $t = 1, \ldots, T$:

1a. learner observes context x_t

- 1b. reward vector $\mathbf{r}_t \in [0, 1]^K$ chosen (but not observed)
 - 2. learner selects action $a_t \in \{1, \ldots, K\}$
 - 3. learner receives observed reward $r_t(a_t)$
- goal: want high total (or average) reward relative to best policy $\pi \in \Pi$
 - i.e., want small regret:



- "no regret" if regret \rightarrow 0 as $\mathcal{T}\rightarrow\infty$

Outline

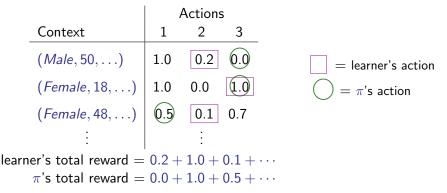
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Starting Point: Full-Information Setting

• full-information setting: same as bandit, but learner can see rewards for all actions



for any π, can compute rewards would have received
average is good estimate of π's expected reward

Follow-the-Leader Algorithm

- at round *t*:
 - find empirically best $\pi \in \Pi \leftarrow$ main challenge
 - use to choose action: $a_t = \pi(x_t)$
- optimal regret: $O\left(\sqrt{\frac{\ln |\Pi|}{T}}\right)$
- to apply, need "oracle" (algorithm/subroutine) for finding best $\pi\in\Pi$ on observed contexts and rewards
 - "arg-max oracle" (aka: ERM oracle, classification oracle, linear oracle, ...)
- same as standard classification learning
- so: if have "good" classification algorithm for Π, can use to find good policy

technique: estimate expected reward of each policy

technique: use existing method ("oracle") to find best policy

- show every policy's empirical average reward close to expected reward
- implies empirically best policy has reward close to truly best policy \Rightarrow regret bound

Non-Stochastic (Adversarial) Setting

- so far, assumed stochastic setting: each (x_t, \mathbf{r}_t) i.i.d.
- not always realistic, e.g.:
 - temporally correlated or drifting data
 - truly adversarial environment (as in game playing)
- non-stochastic (adversarial) setting:
 - contexts x_t and rewards r_t are arbitrary
 - not assumed random
 - possibly selected by adversary
- follow-the-leader does not work here
 - adversary can force very low reward while ensuring one policy gets fairly high reward

Hedge Algorithm

[Littlestone & Warmuth][Freund & Schapire]

- maintain one weight for every $\pi \in \Pi$
- on each round *t*:
 - choose random policy π with probability proportional to weights
 - use action chosen by π
 - increase weight of each policy according to reward it would have received
- yields optimal regret, even in adversarial setting
- but: time/space are linear in $|\Pi|$
 - too slow if $|\Pi|$ gigantic
- applications:
 - game-playing: can use to play/solve games
 - boosting: AdaBoost derived from Hedge

technique: use weighted combination of policies

- keep track of sum of weights of all policies
 - upper bound in terms of reward of algorithm
 - · lower bound in terms of reward of best policy
- combine to get regret bound

Follow-the-Leader versus Hedge

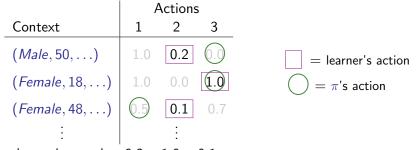
- follow-the-leader:
 - stochastic setting only
 - optimal regret
 - efficient, given access to oracle
- Hedge:
 - non-stochastic setting
 - optimal regret
 - inefficient if $|\Pi|$ huge
- is best of both possible?
 - i.e., no-regret, oracle-efficient algorithm for non-stochastic setting?
 - appears impossible [Hazan & Koren]

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Back to Bandit Setting

only see rewards for actions taken



learner's total reward = $0.2 + 1.0 + 0.1 + \cdots$ π 's total reward = ?? + $1.0 + ?? + \cdots$

- for any policy π , only observe π 's rewards on subset of rounds
- might like to use oracle to find empirically good policy
- problems:
 - only see some rewards
 - observed rewards highly biased (due to skewed choice of actions)

Exploration is Necessary

• e.g.:

- drug A is "pretty good" (cure rate = 60%)
- drug B is "much better" (cure rate = 80%)
- in early trials, by chance, A might appear better than B
- \Rightarrow follow-the-leader can "get stuck" only picking A
 - need exploration!
 - problem even more extreme with more complex policies

e-Greedy/Epoch-Greedy Algorithm

[Langford & Zhang]

[can find with oracle]

(with probability ϵ)

- modified follow-the-leader for bandit stochastic setting
 - explicit exploitation and exploration
- on each round, choose action:
 - according to "best" policy so far (with probability $1-\epsilon)$
 - uniformly at random
- simple and fast (given oracle)
- not optimal regret: C

$$O\left(\left(\frac{K\ln|\Pi|}{T}\right)^{1/3}\right)$$

analysis: similar to follow-the-leader
 technique: explicit exploration via uniform sampling of actions

De-biasing Biased Estimates

- selection bias is major problem
- simple (and old) trick: inverse-propensity weighting
 - say want to estimate E[X] (e.g.: probability unfair coin comes up heads)
 - with probability p: observe X once with probability 1 - p: don't observe X at all!
 - trick: define

$$\hat{X} = \left\{ egin{array}{cc} X/p & ext{if observed} \\ 0 & ext{else} \end{array}
ight.$$

• then $E[\hat{X}] = E[X]$ — unbiased!

 can use to get unbiased estimates for rewards of all actions (not just observed)

- estimates are unbiased done?
- no! variance may be extremely large
- : to get good estimators, must control variance
 - sometimes can do with uniform sampling of actions
 - sometimes need more sophisticated approach

technique: inverse-propensity weighting to get unbiased estimates

Bandits in Non-Stochastic Setting

[Auer, Cesa-Bianchi, Freund & Schapire]

- Exp4: contextual-bandits algorithm for non-stochastic setting
- combines:
 - Hedge
 - uniform sampling of actions
 - inverse-propensity weighting
- optimal regret: $O\left(\sqrt{\frac{K \ln |\Pi|}{T}}\right)$
- analysis: similar to Hedge, but also must account for variance
- but like Hedge: time/space are linear in |Π|

Epoch-Greedy versus Exp4

- epoch-greedy:
 - stochastic setting
 - not optimal regret: O (T^{-1/3})
 - efficient, given access to oracle
- Exp4:
 - non-stochastic setting
 - optimal regret: $O(T^{-1/2})$
 - inefficient if $|\Pi|$ huge
- difference in regret is big!
 - to get regret ε , need $O\left(1/\varepsilon^3\right)$ versus $O\left(1/\varepsilon^2\right)$ trials
- best of both?
 - in stochastic setting, is there an algorithm that is fast (given oracle) and has near optimal regret? yes!

<u>"Mini-Monster" Algorithm (aka: ILOVETOCONBANDITS)</u> [Agarwal, Hsu, Kale, Langford, Li & Schapire]

- apply all preceding techniques
- every round, find weighted combination of policies satisfying explicitly stated properties:
 - 1. low (estimated) regret [exploit] i.e., choose actions think will give high reward
 - 2. low (estimated) variance [explore] i.e., ensure future estimates will be accurate
- can formulate as very large and complex optimization problem
- solve using simple and efficient algorithm, using oracle
 - find violated contraint and fix it repeat until done

Mini-Monster (cont.)

• regret (nearly) optimal:

$$\tilde{O}\left(\sqrt{\frac{K\ln|\Pi|}{T}}\right)$$

• fast! only requires an average of

$$\tilde{O}\left(\sqrt{\frac{\mathcal{K}}{\mathcal{T}\ln|\Pi|}}\right) \ll 1$$

oracle calls per round

 same approach as RandomizedUCB (aka "Monster") but simpler and much faster [Dudík, Hsu, Kale, Karampatziakis, Langford, Reyzin & Zhang]

technique: formulate properties as optimization problem and solve

Proof Ideas

• regret bound:

- regret constraint ensures low regret (if estimates are good enough)
- variance constraint ensures that they actually will be good enough
- efficiency of numerical algorithm:
 - use potential function to measure progress

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Application: Multiworld Testing Decision Service

[Agarwal, Bird, Cozowicz, Hoang, Langford, Lee, Li, Melamed, Oshri, Ribas, Sen, Slivkins]

- unified system for solving contextual-bandit problems
 - general-purpose
 - modular
 - easy to interface with existing systems
 - designed to reduce common errors
- e.g.: deployed to select news articles on MSN homepage
 - no previous learning method had been successful
 - 25% relative lift in click-through rate
 - used now in production (thousands of requests per second)

<u>A Next Step: Contextual Bandits with Underlying State</u> [Jiang, Krishnamurthy, Agarwal, Langford & Schapire]

- decisions made now can significantly impact the future
 - may be underlying state affected by actions
- e.g., medical treatment:
 - see same patient repeatedly
 - state: underlying condition or disease, stage of progression, etc.
 - affected by chosen treatment
- still want to find best policy
 - much harder since choices have impact well into future
 - every policy can define very different sequence of actions
- new exploration algorithm for finding "best" policy
 - assumes feasibility of "value-function approximation"
 - polynomial in new measure of tractability called Bellman rank
 - but: not computationally efficient more to do!

Conclusions

- contextual bandits is a challenging problem, especially if want
 - computational efficiency
 - very large policy space (for highly complex behaviors)
 - optimal statistical performance (regret)
 - adversarial setting
- building up effective methods for meeting these challenges
- theory is indispensible guide, paying off in practice

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